GOVERNMENT DEGREE COLLEGE, RAJAMPETA
(Affiliated to yogi vemana university, Kadapa)
(Re-accredited by NAAC with "B+" Grade in cycle - III)
Rajampet-516115

DEPARTMENT OF MATHEMATICS

Group Discussion on: Groups and Subgroups in Abstract Algebra

Date & Time: 13-03-2026 11.30 AM

Participants: I Year B.Sc. Mathematics students and faculty members

1. Aim

To deepen the conceptual understanding of algebraic structures, specifically groups and subgroups, among students and faculty, and to explore their foundational role in modern mathematics through collaborative discussion.

2. Objectives

- To review the **definition and axioms of a group** (closure, associativity, identity, inverse).
- To distinguish between **different types of groups** (finite, infinite, abelian, non-abelian, cyclic, permutation groups).
- To examine the **criteria for subgroups** (one-step, two-step, and finite subgroup tests).
- To identify **examples and counterexamples** of subgroups in standard groups like $\mathbb{Z}, \mathbb{Q}^*, S_n, D_n$, and matrix groups.
- To analyze **properties of subgroups** including cosets, Lagrange's theorem, and normal subgroups (as a bridge to quotient groups).
- To encourage **peer learning and problem-solving** through group discussion.

3. Topics for Discussion

A. Fundamental Concepts

1. Definition of a group with illustrative examples.
2. Abelian vs. non-abelian groups.
3. Order of a group and order of an element.

B. Subgroup Theory

1. Definition and necessary/sufficient conditions for a subset to be a subgroup.
2. Trivial and proper subgroups.

3. Subgroup generated by a subset – cyclic subgroups.
4. Intersection and union of subgroups (union is not always a subgroup – discussion).
5. Subgroup lattice of small groups (e.g., $\mathbb{Z}_6, S_3$).

C. Important Examples and Counterexamples

1. Subgroups of $\mathbb{Z}$ under addition: $n\mathbb{Z}$.
2. Subgroups of S_3 and S_4.
3. $GL(n,\mathbb{R})$, $SL(n,\mathbb{R})$, $O(n)$ as subgroups.
4. Center of a group and its subgroup property.
5. Normalizer and centralizer – are they always subgroups?

D. Key Theorems (to be discussed conceptually)

1. Lagrange's theorem and its converse (counterexample: A_4 has no subgroup of order 6).
2. Cauchy's theorem for finite groups (motivation).
3. Normal subgroups as kernels of homomorphisms.

E. Problem-Solving Scenarios

- Given a group G and a subset H, determine if H is a subgroup.
- Find all subgroups of a cyclic group.
- Show that the intersection of two subgroups is a subgroup; give an example where union fails.
- Identify whether a given subgroup is normal.

4. Mode of Discussion

- **Moderator:** Sri V.K. Mastan
- **Format:** Students divided into small groups (4–5 members) for topic-wise discussion, followed by a plenary session.
- **Outcome:** Each group presents key findings, doubts, and examples. A summary report will be prepared and added to the department's academic repository.

5. Expected Outcomes

- Students will be able to **rigorously prove** if a given set forms a group or subgroup.
- Improved ability to **construct counterexamples**.

- Better preparedness for competitive exams (e.g., CSIR-NET, SLET, IIT-JAM) that emphasize group theory.
- Enhanced collaborative learning and communication of mathematical ideas.

Group Discussion

Topic:- Groups & sub groups

Name of the student	Signature
1) S. Meheth Fathima	S. Meheth Fathima
2) S. Madiha	S. Madiha
3) B. Sivaparvathi	B. Sivaparvathi
(4) S. Manasa	S. Manasa.
(5) D. Divya	D. Divya
(6) M. Sisira	M. Sisira
7) R. Vani	R. Vani
8) S. Navya	S. Navya
9) P. Malleswari	P. Malleswari
S. Manasa	S. Manasa
10) P. Sukanya	P. Sukanya.
11) P. Sravani	P. Sravani
12) M. Anitha	M. Anitha
13) E. Sireesha	E. Sireesha

M. Jayal[illegible]

Signature of the In-Charge